1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 04 (Physics Part -2, Class XII)
Module Name/Title	Unit-06, Module-06: Refraction at Spherical Surfaces Chapter-09: Ray Optics
Module Id	leph_20906_eContent
Pre-requisites	Reflection, Refraction, laws of reflection, laws of refraction. curved surfaces, radius of curvature , center of curvature, real and virtual image , mirror formula relating object distance image distance and focal length .
Objectives	After going through this module, the learner will be able to: • Draw ray diagrams to show refraction at spherical surfaces • Appreciate the need for Sign convention • Predict and understand refraction by a lens • Deduce lens makers formula and describe power of lens • Define Foci, focal plane, focal length, optical center, principal axis • Explain refraction through combination of thin lenses in contact
Keywords	Lens formula , lens makers formula, sign convention, power of lens

2. Development Team

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Ray optics Reflection of light; spherical mirrors; mirror formula; refraction of light; total internal reflection and its applications; optical; fibers; refraction at spherical surfaces; lenses; thin lens formula; lens maker's formula; magnification, power of a lens; combination of thin lenses in contact; refraction and dispersion of light through a prism.

Scattering of light – blue color of sky and reddish appearance of the sun at sunrise and sunset

Optical instruments – microscopes and astronomical telescopes (refracting and reflecting) and their magnifying powers

Chapter 10 Wave Optics

Wave optics: wave front and Huygens's principle, reflection and refraction of plane wave at a plane surface using wave fronts. proof of laws of reflection and refraction using Huygens's principle. Interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light; diffraction due to a single slit width of central maximum; resolving power of microscope and astronomical telescope. Polarisation, plane polarised light, Malus's law, Brewster's law, uses of plane polarised light and polaroid.

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MODULE 6

3. WORDS YOU MUST KNOW

Let us remember the words and the concepts we have been using in the study of this module:

- **Light:** Light is a form of energy which gives the sensation of vision when it falls on the retina of the eye.
- **Ray of light:** The straight line path along which light travels is called a ray of light. Light rays start from each point of a source and travel along straight line until they strike an object or a surface separating two media.
- **Beam of light:** A group of rays of light is called a beam of light.
- **Parallel beam of light**: If all the rays of light in the group are parallel to each other then the beam is said to be a parallel beam of light.
- **Converging beam of light**: If the rays of light in the group come closer to each other i.e. converge to a point, then the beam is said to be a converging beam of light.
- **Diverging beam of light**: If the rays of light in the group move away from each other i.e. diverge, then the beam is said to be a diverging beam of light.
- **Transparent medium:** A medium through which light can pass freely over large distance is called a transparent medium. Glass and still water are some examples of transparent objects
- **Opaque medium:** A medium through which light cannot pass is called an opaque medium. Wood and metals are some examples of opaque objects.
- **Real image:** If the rays of light after reflection from a mirror actually meet at a point i. e. the reflected beam is a converging beam, then the image is said to be real image.
- **Virtual image:** If the rays of light after reflection from a mirror do not actually meet at a point but meet on producing backwards i.e. the reflected beam is a diverging beam, then the image is said to be a virtual image.
- **Refractive index** = n =speed of light in vacuum / speed of light in the medium
- **Relative refractive index:-** Consider light going from medium 1 to medium 2

  Then refractive index of medium 2 with respect to medium 1 is

$$n_{21} = (n_2/n_1) = v_1 / v_2$$

- **Laws of Refraction of light :-**

i. The incident ray, the refracted ray and the normal at the point of incidence, all lie in the same plane.

ii. The ratio of sine of the angle of incidence to the sine of the angle of refraction *r, for two media* is constant for a given wavelength of light and is equal to the refractive index of the second medium with respect to first medium.

- **Critical angle:-** That angle of incidence in denser medium for which the refracted ray just grazes the interface of two media is called the critical angle
- **Total internal reflection: -**

  The phenomenon in which a ray of light travelling from a denser medium to rarer medium at an angle of incidence greater than critical angle is totally reflected back into the same medium is called total internal reflection.

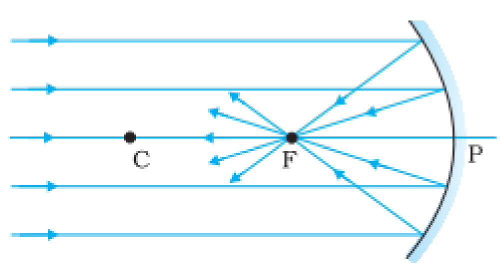
- **Conditions for total internal reflection:**

1) Light must travel from optically denser medium to optically rarer medium.
2) Angle of incidence must be more than critical angle

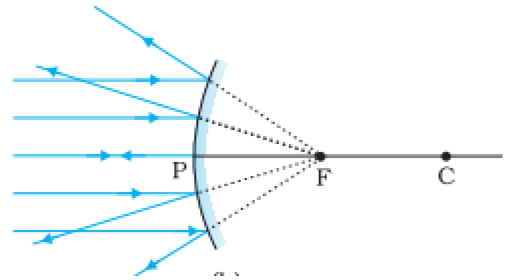
- **Relation between refractive index and critical angle:**

$$n = 1/\sin i_c$$

- **Convex** mirror: if the reflecting surface convex
- **Aperture** or the lateral size of **the mirror or lens**
- **Pole or vertex** center of the mirror
- **Focus** The reflected rays converge at a point F on the principal axis of a concave mirror



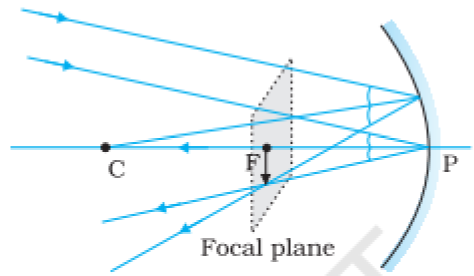
For a convex mirror, the reflected rays appear to diverge from a point F on its principal axis



The point F is called the principal focus

Focal length the distance from the pole/center of the mirror to the focus is the focal length.

Focal plane the plane perpendicular to the principal axis containing the focus is called the focal plane



Real image If rays emanating from a point actually meet at another point after reflection and/or refraction, that point is called the image of the first point. The image is real if the rays actually converge to the point.

Virtual image virtual if the rays do not actually meet but appear to diverge from the point when produced backwards.

An image is thus a point-to-point correspondence with the object established through reflection and/or refraction

Object distance (u) – Distance of the object from the pole

Image distance (v) - Distance of the image from the pole

Linear Magnification =$m = \frac{\text{size of image}}{\text{size of object}} = -\frac{v}{u}$

Angular magnification $= \frac{\text{angle subtented by the image}}{\text{angle subtented by the object if placed at the same position as the image}}$

Rectangular glass slab: A parallel sided glass cuboid

Equilateral glass prism: A glass prism with refracting surfaces at 60^0

Lens: A thin lens is a transparent optical medium bounded by two surfaces; at least one of which should be spherical,

4. INTRODUCTION

Nature has endowed the human eye (retina) with the sensitivity to detect electromagnetic waves within a small range of the electromagnetic spectrum. Electromagnetic radiation belonging to this region of the spectrum (wavelength of about 400 nm to 750 nm) is called light. It is mainly through light and the sense of vision that we know and interpret the world around us.

There are two things that we can intuitively mention about light from common experience.

First, that it travels with enormous speed and

Second, that it travels in a straight line.

It took some time for people to realize that the **speed of light is finite and measurable.**

Its presently accepted value in vacuum is $c = 2.99792458 \times 10^8$ m s^{-1}.

For many purposes, it suffices to take $c = 3 \times 10^8$ m s^{-1}.

The speed of light in vacuum is the highest speed attainable in nature.

We are familiar with the **laws of reflection**. The angle of reflection (i.e. the angle between reflected ray and the normal to the reflecting surface or the mirror) equals the angle of incidence (angle between incident ray and the normal). Also that the incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane

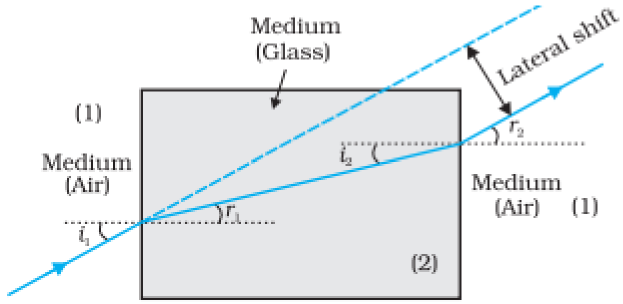
These laws are valid at each point on any reflecting surface whether plane or curved. The normal in this case is to be taken as normal to the tangent to surface at the point of incidence. That is, the normal is along the radius, the line joining the centre of curvature of the mirror to the point of incidence.

You may recall laws of refraction. Refractive index and Snell's law

We know that, when a beam of light encounters another transparent medium, a part of light gets reflected back into the first medium while the rest enters the other.

You call also recall some elementary results based on the above laws for light.

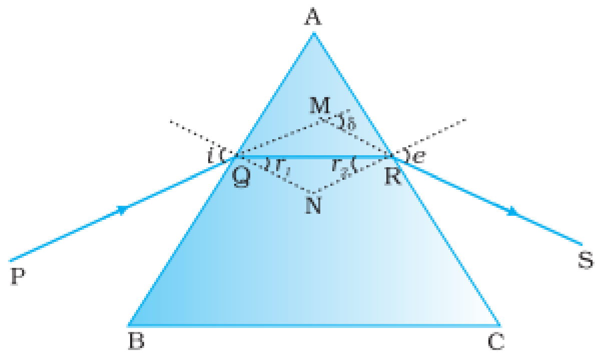
For a rectangular slab, refraction takes place at two interfaces (air-glass and glass-air). It is easily seen from that $r_2 = i_1$, i.e**., the emergent ray is parallel to the incident ray**—there is no deviation, but it does suffer lateral displacement/ shift with respect to the incident ray



Lateral shift of a ray refracted through a parallel-sided slab.

For an equilateral prism

For the passage of light through a triangular prism ABC, the angles of incidence and refraction at the first face AB are i and r_1, while the angle of incidence (from glass to air) at the second face AC is r_2 and the angle of refraction or emergence e.



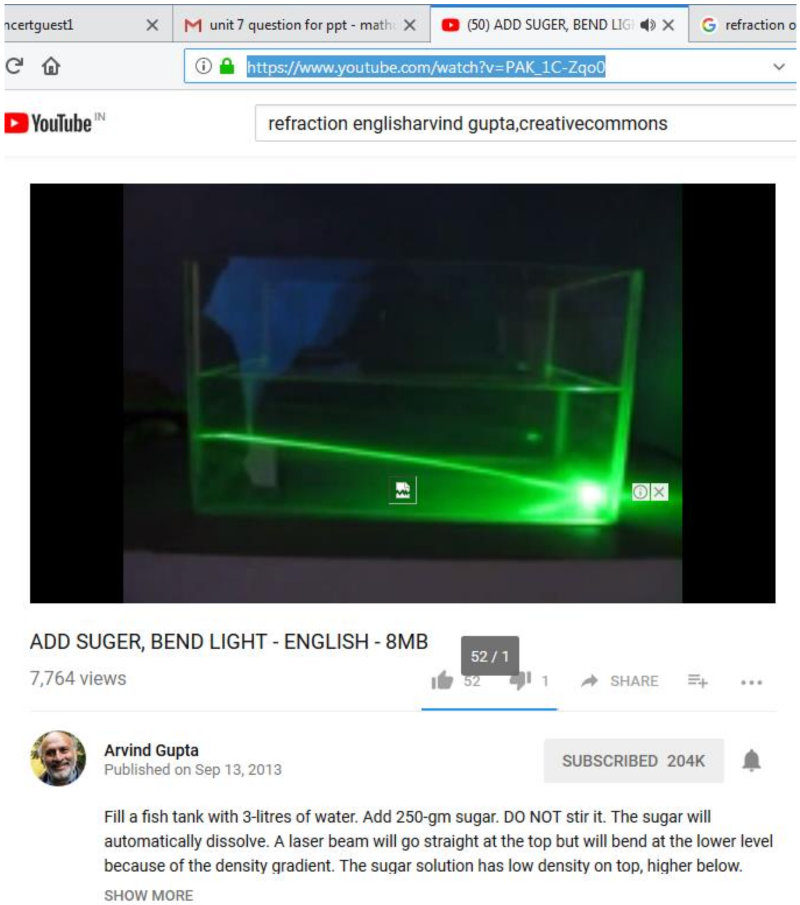
A ray of light passing through a triangular glass prism.

The angle between the emergent ray RS and the direction of the incident ray PQ is called the angle of deviation, δ. The angle of deviation depends on the angle of incidence i and the angle of prism BAC (A) and the material of the prism

In all the cases we considered the medium in which light travelled was homogeneous, this means medium having uniform optical density.

You may like to do this experiment

https://www.youtube.com/watch?v=PAK_1C-Zqo0



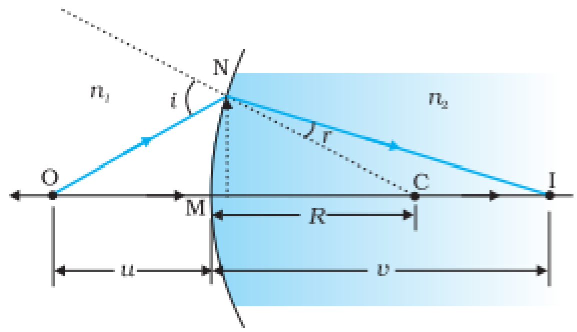
5. REFRACTION AT SPHERICAL SURFACES

We have so far considered refraction at a plane interface.

We shall now consider refraction at a **spherical interface between two transparent media**.

An infinitesimal part of a spherical surface can be regarded as planar and the same laws of refraction can be applied at every point on the surface.

Just as for reflection by a spherical mirror, the normal at the point of incidence is perpendicular to the tangent plane to the spherical surface at that point and, therefore, passes through its centre of curvature.



Refraction at a spherical surface separating two media

Shows the geometry of formation of image I of an object O on the principal axis of a spherical surface with centre of curvature C, and radius of curvature R.

The rays are incident from a medium of refractive index n_1, to another of refractive index n_2. As before, we take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made. In particular, NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis. We have, for small angles,

$$\tan\angle NOM = \frac{MN}{OM}$$

$$\tan\angle NCM = \frac{MN}{MC}$$

$$\tan\angle NIM = \frac{MN}{MI}$$

Now, for ΔNOC, i is the exterior angle. Therefore, i = ∠NOM + ∠NCM

$$\mathrm{i} = \frac{\mathrm{MN}}{\mathrm{OM}} + \frac{\mathrm{MN}}{\mathrm{MC}}$$

Similarly

$$r = \frac{MN}{MC} - \frac{MN}{MI}$$

Using Snell's law

$$\boldsymbol{n_1 \sin i = n_2 sinr}$$

Or for **small angles $\boldsymbol{n_1 i = n_2 r}$**

Substituting i and r, we get

$$\boldsymbol{n_1}\left(\frac{MN}{OM}+\frac{MN}{MC}\right)=\boldsymbol{n_2}\left(\frac{MN}{MC}-\frac{MN}{MI}\right)$$

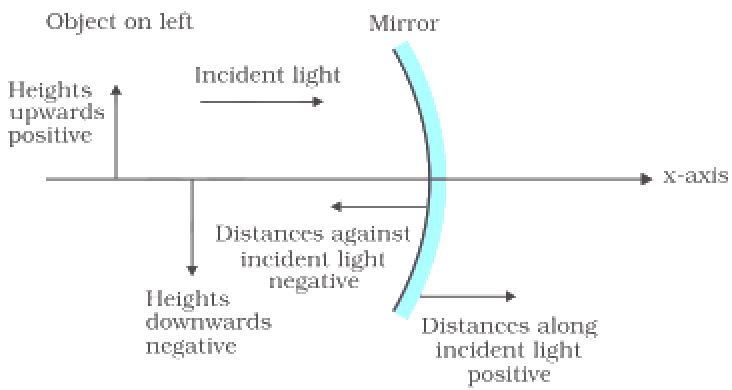
$$\frac{\mathbf{n_1}}{\mathbf{OM}}+\frac{\mathbf{n_2}}{\mathbf{MI}}=\frac{\mathbf{n_2}-\mathbf{n_1}}{\mathbf{MC}}$$

Here, OM, MI and MC represent magnitudes of distances.

Applying the Cartesian sign convention,

SIGN CONVENTION

To derive the relevant formulae for reflection by spherical mirrors and refraction by spherical lenses, we must first adopt a sign convention for measuring distances. In this book, we shall follow the **Cartesian sign convention**. According to this convention,



The Cartesian Sign Convention.

- All distances are measured from the pole of the mirror or the optical centre of the lens.
- The distances measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of incident light are taken as negative
- The heights measured upwards with respect to x-axis and normal to the principal axis (x-axis) of the mirror/ lens are taken as positive
- The heights measured downwards are taken as negative.
- With a common accepted convention, it turns out that a single formula for spherical mirrors and a single formula for spherical lenses can handle all different cases.

Here, OM, MI and MC represent magnitudes of distances. Applying the Cartesian sign convention,

OM = –u, MI = +v, MC = +R

Substituting we get

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

This equation gives us a **relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface**.

It holds for any curved spherical surface.

The image may be real or virtual depending upon the nature of the surface vis concave or convex as we have considered.

EXAMPLE

Consider the following situations and predict the relation according to whether the image is real or virtual

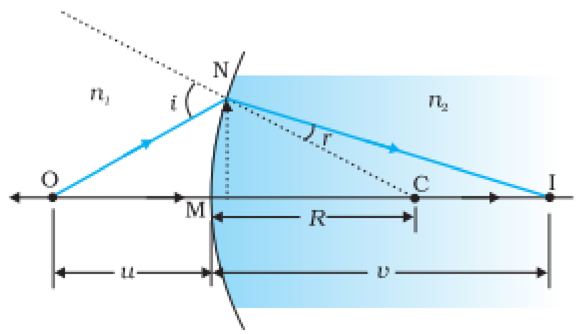
i) **If light travels from optically rarer medium to optically denser medium and the image formed is real**
ii) **If light travels from optically rarer medium to optically denser medium and the image formed is virtual**
iii) **If light travels from optically denser medium to optically rarer medium and the image formed is virtual**
iv) **If light travels from optically denser medium to optically rarer medium and the image formed is real**

SOLUTION

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

(i) If light travels from optically rarer medium to optically denser medium and the image formed is real

Object distance OM =-u
Image distance MI =+v
Radius of curvature MC=+R
Image is real



(ii) If light travels from optically rarer medium to optically denser medium and the image formed is virtual

Object distance OM = - u

Image distance MI = - v

Radius of curvature MC= - R

Image is virtual

$$\frac{\mathbf{n_2}}{-\mathbf{v}} - \frac{\mathbf{n_1}}{-\mathbf{u}} = \frac{\mathbf{n_2} - \mathbf{n_1}}{-\mathbf{R}}$$

(iii) If light travels from optically denser medium to optically rarer medium and the image formed is virtual

Object distance OM = -u

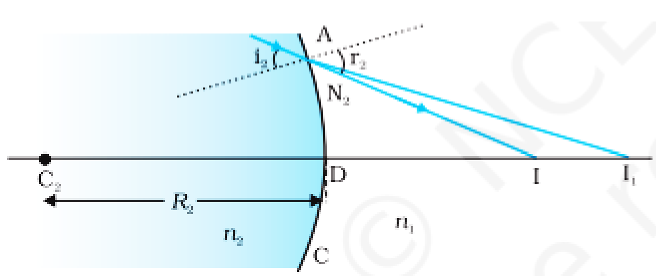
Image distance MI = -v

Radius of curvature MC= -R

Image is virtual

$$\frac{\mathbf{n_2}}{-\mathbf{v}} - \frac{\mathbf{n_1}}{-\mathbf{u}} = \frac{\mathbf{n_2} - \mathbf{n_1}}{-\mathbf{R}}$$

(iv) If light travels from optically denser medium to optically rarer medium and the image formed is real



Object distance OM = -u

Image distance MI = +v

Radius of curvature MC= +R

Image is virtual

$$\frac{\mathbf{n_2}}{\mathbf{v}} - \frac{\mathbf{n_1}}{-\mathbf{u}} = \frac{\mathbf{n_2 - n_1}}{\mathbf{R}}$$

The factor

$$\frac{\mathbf{n_2 - n_1}}{\mathbf{R}}$$

Is the factor which indicates the ability of the spherical refracting surface to converge or diverge the rays

EXAMPLE

Light from a point source in air falls on a spherical glass surface (n = 1.5 and radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm.

At what position the image is formed?

SOLUTION

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Assuming speed of light in air to be same as that in vacuum

Here u = – 100 cm, v =? R = + 20 cm, n_1 = 1, and n_2 = 1.5. We then have

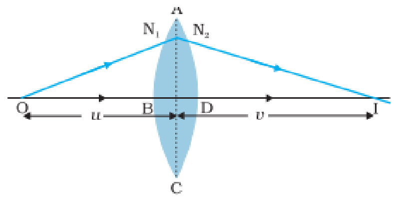
$$\frac{1.5}{v} + \frac{1}{100} = \frac{0.5}{20}$$

V= + 100cm

The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

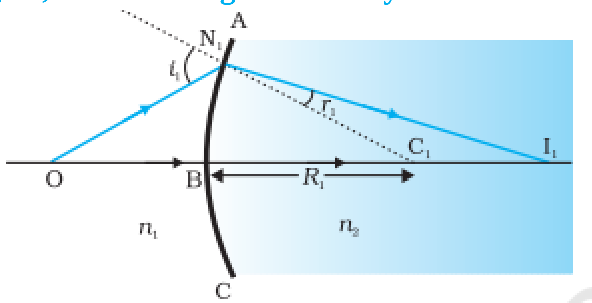
6. REFRACTION BY A LENS

Lens is a transparent material bounded by spherical surfaces. Glass lens is solid while water lens is made if liquid



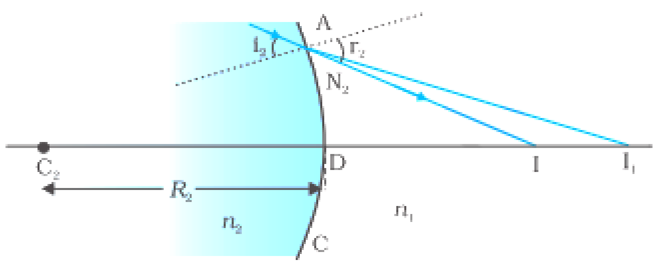
(a)

a) The position of object, and the image formed by a double convex lens



(b)

b) Refraction at the first spherical surface



(c)

c) Refraction at the second spherical surface.

Figur**e** shows the geometry of image formation by a double convex lens.

The image formation can be seen in terms of two steps:

(i) The first refracting surface forms the image I_1 of the object O

(ii) The image I_1 acts as a virtual object for the second surface that forms the image at I

Applying

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

to the first interface ABC, we get

$$\frac{n_1}{OB} + \frac{n_1}{BI_1} = \frac{n_2 - n_1}{BC_1}$$

A similar procedure applied to the second interface* ADC gives,

$$-\frac{n_2}{DI_1}+\frac{n_1}{DI}=\frac{n_2-n_1}{DC_2}$$

Note

That now the refractive index of the medium on the right side of ADC is n_1 while on its left it is n_2.

Further DI_1 is negative as the distance is measured against the direction of incident light

For a **thin lens**, $BI_1 = DI_1$. Adding the above two equations, we get

$$\frac{\mathrm{n}_1}{\mathrm{OB}}+\frac{\mathrm{n}_1}{\mathrm{DI}}=\mathrm{n}_2-\mathrm{n}_1\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right)$$

Suppose the object is at infinity, i.e., OB → ∞ and DI = f, we will then get

$$\frac{n_1}{f}=(n_2-n_1)\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right)$$

The point where image of an object placed at infinity is formed is called the focus F, of the lens and the distance f gives its focal length.

A lens has two foci, F and F′, on either side of it

By the sign convention,

$BC_1 = + R_1$,

$DC_2 = –R_2$

So we can write

$$\frac{\mathbf{n_1}}{\mathbf{f}}=(\mathbf{n_2}-\mathbf{n_1})\left(\frac{\mathbf{1}}{\mathbf{R_1}}-\frac{\mathbf{1}}{\mathbf{R_2}}\right)$$

Or

$$\frac{\mathbf{1}}{\mathbf{f}}=\frac{(n_2-n_1)}{n_1}\left(\frac{\mathbf{1}}{\mathbf{R_1}}-\frac{\mathbf{1}}{\mathbf{R_2}}\right)$$

This is called the lens makers formula

We also get another important result from

$$\frac{n_1}{-u} + \frac{n_1}{v} = n_2 - n_1\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ **is the familiar thin lens formula?**

Though we derived it for a real image formed by a convex lens, the formula is valid for both convex as well as concave lenses and for both real and virtual images.

It is worth mentioning that the two foci, F and F′, of a double convex or concave lens are equidistant from the optical centre.

The focus on the side of the (original) source of light is called the first focal point, whereas the other is called the second focal point.

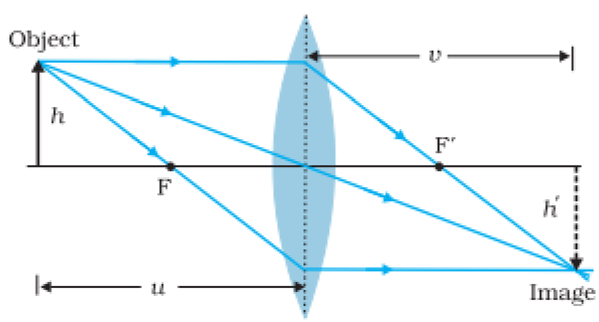
To find the image of an object by a lens, we can, in principle, take any two rays emanating from a point on an object; trace their paths using the laws of refraction and find the point where the refracted rays meet (or appear to meet).

In practice, however, it is convenient to choose any two of the following rays:

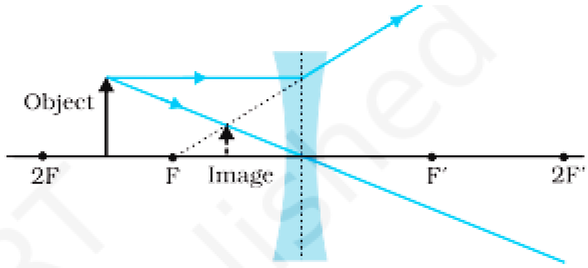
(i) **A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the second principal focus F′ (in a convex lens) or appears to diverge (in a concave lens) from the first principal focus F**

(ii) **A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction.**

(iii) **A ray of light passing through the first principal focus (for a convex lens) or appearing to meet at it (for a concave lens) emerges parallel to the principal axis after refraction.**



(a)



(b)

(a) and (b) illustrate these rules for a convex and a concave lens, respectively.

You should practice drawing similar ray diagrams for different positions of the object with respect to the lens and also verify that the lens formula,

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ **holds good for all cases.**

Here again it must be remembered that each point on an object gives out infinite number of rays. All these rays will pass through the same image point after refraction at the lens.

Magnification (m) produced by a lens is defined, like that for a mirror, as the ratio of the size of the image to that of the object.

$$m = \frac{\text{size of image}}{\text{size of object}} = \frac{v}{u}$$

EXAMPLE

A magician during a show makes a glass lens with n = 1.47 disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

SOLUTION

The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$.

This gives $1/f = 0$ or $f \rightarrow \infty$.

The lens in the liquid will act like a plane sheet of glass.

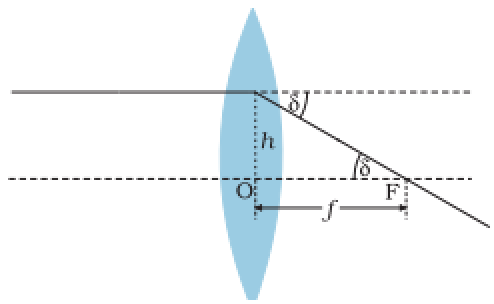
No, the liquid is not water. It could be glycerin.

POWER OF A LENS

Power of a lens is a measure of the convergence or divergence, which a lens introduces in the light falling on it.

Clearly, a lens of shorter focal length bends the incident light more, while converging it in case of a convex lens and diverging it in case of a concave lens.

The power P of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre.



$$\tan\delta = \frac{h}{f} \quad if\ h = 1\ or\ \delta = \frac{1}{f}$$

$$P = \frac{1}{f}$$

The SI unit for power of a lens is dioptre (D): 1D = 1m^{-1}.

The power of a lens of focal length of 1 metre is one dioptre.

Power of a lens is **positive** for a converging lens and **negative** for a diverging lens.

Thus, when an **optician prescribes** a corrective lens of power + 2.5 D, the required lens is a convex lens of focal length + 40 cm.

A lens of power of – 4.0 D means a concave lens of focal length – 25 cm.

The lens maker would use the lens maker's formula to grind a piece of good quality glass for required radius of curvature making it of desired focal length.

EXAMPLE

(i) **If f = 0.5 m for a glass lens, what is the power of the lens?**

(ii) **The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass?**

(iii) **A convex lens has 20 cm focal length in air. What is focal length in water? (Refractive index of air-water = 1.33, refractive index for air-glass = 1.5.)**

SOLUTION

(i) Power = +2 dioptre.

(ii) Here, we have f = +12 cm, R_1 = +10 cm, R_2 = –15 cm.
Refractive index of air is taken as unity.
We use the lens formula. The sign convention has to be applied for f, R_1 and R_2 . Substituting the values, we have

$$\frac{1}{12} = (n-1)\left(\frac{1}{10} - \frac{1}{-15}\right)$$

n=1.5

(iii) For a glass lens in air, n_2 = 1.5, n_1 = 1, f = +20 cm.
Hence, the lens formula gives

$$\frac{1}{20} = 0.5\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For the same glass lens in water, n2 = 1.5, n1 = 1.33.

Therefore

$$\frac{1.33}{f} = (1.5 - 1.33)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Combining these two equations, we find f = + 78.2 cm.

EXAMPLE

An object is packed in front of convex lens made of glass how does the image distance vary if the refractive index of the medium is increased in such a way that it remains less than the glass

SOLUTION

$$\frac{1}{f} = \frac{1}{\mathrm{v}} - \frac{1}{u} = (\mathrm{n}_2 - \mathrm{n}_1)\left(\frac{1}{\mathrm{R}_1} - \frac{1}{\mathrm{R}_2}\right)$$

n_1 =1 if n_1 increases 1/f, decreases or f increases therefore v increases

COMBINATION OF THIN LENSES IN CONTACT

Consider two lenses A and B of focal length f_1 and f_2 placed in contact with each other. Let the object be placed at a point O beyond the focus of the first lens A, see figure

The first lens produces an image at I_1. Since image I_1 is real, it serves as a virtual object for the second lens B, producing the final image at I.

It must, however, be borne in mind that formation of image by the first lens is presumed only to facilitate determination of the position of the final image. In fact, the direction of rays emerging from the first lens gets modified in accordance with the angle at which they strike the second lens. Since the lenses are thin, we assume the optical centres of the lenses to be coincident.

Let this central point be denoted by P. For the image formed by the first lens A, we get

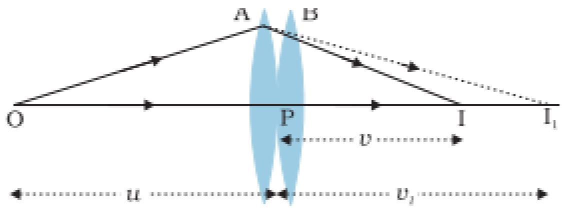


Image formation by a combination of two thin lenses in contact.

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

For the image formed by the second lens B, we get

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$$

Adding the two equations, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

If the two lens-system is regarded as equivalent to a single lens of focal length f, we have

$$\frac{1}{f} = \frac{1}{\mathrm{v}} - \frac{1}{u}$$

Or

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

The derivation is valid for any number of thin lenses in contact. If several thin lenses of focal length f_1, f_2, f_3, ... are in contact, the effective focal length of their combination is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \cdots.$$

$P = P_1 + P_2 + P_3 + \ldots$ where P is the net power

Note that the sum is an algebraic sum of individual powers, so some of the terms on the right side may be positive (for convex lenses) and some negative (for concave lenses).

Combination of lenses helps to obtain diverging or converging lenses of desired magnification.

It also enhances sharpness of the image. Since the image formed by the first lens becomes the object for the second, this effectively implies that the total magnification m of the combination is a product of magnification (m_1, m_2, m_3,...) of individual lenses $m = m_1 m_2 m_3 ...$

TRY THESE

1. **Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20cm?**

2. **A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20cm, and (b) a concave lens of focal length 16cm?**

3. **An object of size 3.0cm is placed 14cm in front of a concave lens of focal length 21cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?**

4. **What is the focal length of a convex lens of focal length 30cm in contact with a concave lens of focal length 20cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.**

SUMMARY

- An infinitesimal part of a spherical surface can be regarded as planar and the same laws of refraction can be applied at every point on the surface. Just as for reflection by a spherical mirror, the normal at the point of incidence is perpendicular to the tangent plane to the spherical surface at that point and, therefore, passes through its centre of curvature.

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

  This equation gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface

- For refraction through a spherical interface (from medium 1 to 2 of refractive index n_1 and n_2 , respectively) Lens maker's formula

$$\frac{\mathbf{n_1}}{\mathbf{f}} = (\mathbf{n_2} - \mathbf{n_1})\left(\frac{\mathbf{1}}{\mathbf{R_1}} - \frac{\mathbf{1}}{\mathbf{R_2}}\right)$$

or

$$\frac{\mathbf{1}}{\mathbf{f}} = \frac{(\boldsymbol{n_2} - \boldsymbol{n_1})}{\boldsymbol{n_1}}\left(\frac{\mathbf{1}}{\mathbf{R_1}} - \frac{\mathbf{1}}{\mathbf{R_2}}\right)$$

R_1 and R_2 are the radii of curvature of the lens surfaces. f is positive for a converging lens; f is negative for a diverging lens

- Thin lens formula

$$\frac{1}{f} = \frac{1}{\mathrm{v}} - \frac{1}{u}$$

- The power of a lens P = 1/f. The SI unit for power of a lens is dioptre (D): 1 D = 1 m^{-1}.
- If several thin lenses of focal length f_1 , f_2 , f_3 ,... are in contact, the effective focal length of their combination is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \cdots.$$

- The total power of a combination of several lenses is

$P = P_1 + P_2 + P_3 + \ldots$ where P is the net power